HOMEWORK 1. DUE, IN CLASS, WED JAN 13

1. WE USE THE "BETA – GAMMA CALCULUS" SO OFTEN, WE'D LIKE TO MAKE SURE YOU KNOW HOW IT WORKS. THE GAMMA DENSITY ON $[0,\infty)$ IS $e^{-x}x^{a-1}/\Gamma(a)$ FOR $a>0$, A FIXED PARAMETER. THE BETA DENSITY ON $[0,1]$ IS $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ FOR $\alpha, \beta>0$. LET X, Y, Z BE INDEPENDENT RANDOM VARIABLES HAVING GAMMA DENSITIES WITH RESPECTIVE PARAMETERS α, β, γ. SHOW THAT

$$X/(X+Y), \quad X+Y/(X+Y+Z) \quad \text{AND} \quad X+Y+Z$$

ARE INDEPENDENT $\beta(\alpha,\beta)$, $\beta(\alpha+\beta,\gamma)$, $\text{GAMMA}(\alpha+\beta+\gamma)$ DISTRIBUTED. CAN YOU GIVE A CONCEPTUAL REASON FOR THE INDEPENDENCE?

2. AN URN CONTAINS ONE RED, ONE WHITE AND ONE BLUE BALL. EACH TIME, A BALL IS CHOSEN AT RANDOM (UNIFORMLY) FROM THE URN AND REPLACED, TOGETHER WITH A NEW BALL OF THE SAME COLOR. LET $X_1, X_2, X_3 \cdots$ DENOTE THE COLORS DRAWN ON TRIALS $1, 2, 3, \cdots$. CONSIDER ANOTHER PROCESS TAKING VALUES IN $\{R, W, B\}$: PICK $(p_1, p_2, p_3) \geq 0$, $p_1+p_2+p_3=1$ FROM THE UNIFORM DISTRIBUTION ON THE SIMPLEX. FIX THIS AND DRAW REPEATEDLY AND INDEPENDENTLY FROM THIS DISTRIBUTION REPORTING R, W, OR B AS YOU DRAW 1, 2 OR 3. LET $Y_1, Y_2, Y_3, \cdots$ BE THE RESULTING PROCESS. PROVE THAT, FOR ANY n

$$P(Y_1=y_1, Y_2=y_2 \cdots Y_n=y_n) = P(X_1=y_1, X_2=y_2, \cdots, X_n=y_n).$$

3. LET $\alpha_1, \alpha_2, \alpha_3, \cdots$ BE POSITIVE NUMBERS WITH $A=\sum_{i=1}^{\infty} \alpha_i < \infty$. LET Z_i BE INDEPENDENT GAMMA RANDOM VARIABLES WITH PARAMETERS α_i. SHOW THAT $S=\sum_{i=1}^{\infty} Z_i$ IS FINITE 'ALMOST SURELY'. DEFINE A RANDOM PROBABILITY $Y=(Y_1, Y_2, \cdots)$ BY $Y_i = Z_i/S$. THE JOINT LAW OF THE Y_i IS CALLED D_α (DIRICHLET(α)). HERE IS ANOTHER CONSTRUCTION:

$$\text{LET } W_1 \sim \text{BETA}(\alpha_1, A-\alpha_1),\ W_2 \sim \text{BETA}(\alpha_2, A-\alpha_1-\alpha_2),\ W_3 \sim \text{BETA}(\alpha_2, A-\alpha_1-\alpha_2-\alpha_3), \cdots$$

ALL INDEPENDENT, FORM AN INFINITE 'RANDOM PROBABILITY' BY 'STICK BREAKING'

$$P_0 = W_1,\ P_1 = W_2(1-P_0),\ P_2 = W_3(1-P_0-P_1), \cdots$$

SHOW THAT $P=(P_0, P_1, P_2 \cdots)$ HAS A D_α DISTRIBUTION.

NOTES ON BAYESIAN BOOKS.

THERE ARE NOW MANY REASONABLE BAYESIAN BOOKS; I PUT ONE OF THEM WHICH SEEMS CURRENT & GOOD ON RESERVE FOR STATISTICS 370

1 JEFF GILL (2008) BAYESIAN METHODS CHAPMAN AND HALL. THIS HAS A SOCIAL SCIENCE PERSPECTIVE BUT HAS GOOD COVERAGE OF MATH AND COMPUTING BACKGROUND

2. A USEFUL BOOK WHICH YOU CAN DOWN-LOAD FROM THE WEB IS E.T. JAYNES (1995) PROBABILITY THEORY: THE LOGIC OF SCIENCE; http://omega.albany.edu:8008/JAYNES BOOK.html. THIS WAS MY FIRST INTRODUCTION TO BAYES; ITS VERY DOWN TO EARTH AND WORTH WHILE. SEE ALSO, DAVID MACKAY (2003). INFORMATION THEORY, INFERENCE AND LEARNING ALGORITHMS.

3,4 THE 'STANDARD' MODERN BOOKS ARE, BERGER, J. (1985). STATISTICAL DECISION THEORY AND BAYESIAN ANALYSIS 2nd ED. SPRINGER; AND BERNARDO, J. AND SMITH, A. (1994). BAYESIAN THEORY

5,6. TWO CLASSICS: JEFFREYS, H. (1961). THEORY OF PROBABILITY 3rd ED. OXFORD AND LINDLEY, D.V. (1965). INTRODUCTION TO PROBABILITY AND STATISTICS FROM A BAYESIAN VIEWPOINT CAMBRIDGE. (THE FIRST COURSE I EVER TAUGHT WAS OUT OF LINDLEY, ITS STILL GOOD AND JEFFREYS BOOK IS GREAT)

7,8. TWO MODERN BOOKS: GELMAN, A, CARLIN, B, STERN, H. AND RUBIN D. (~~1995~~ 2013). BAYESIAN DATA ANALYSIS CHAPMAN HALL, LEONARD, T. AND HSU, J. (1999). BAYESIAN METHODS OXFORD. BOTH ARE FULL OF PRACTICAL EXAMPLES IN REAL PROBLEMS.

9. A VERY GOOD BOOK ON MONTE-CARLO METHODS FOR BAYESIAN ANALYSIS IS Chen, H. Shao, Q, Ibrahim J. (2000). MONTE CARLO METHODS IN BAYESIAN COMPUTATION SPRINGER;

QA 279.5.

OUR LIBRARY HAS A BAYESIAN SECTION, SEE IN PARTICULAR THE SERIES 'STUDIES IN BAYESIAN APPLICATION SERIES R. KASS ED. ALSO, THE SERIES OF CONFERENCE PROCEEDINGS BAYESIAN STATISTICS I — VII ED BY. JOSE BERNARDO ET AL, FOR MANY APPLICATIONS + CURRENT RESEARCH

MORE COMING!