STATS 370 Problem Set 3

Problem 1

Jeffreys' prior: Consider the simple ordinary linear regression

$$y_i = \mu + x_i\beta + \epsilon_i$$

for $i = 1, 2, \ldots, n$, where the predictors x_i are fixed, and ϵ_i are i.i.d. normal with mean 0 and variance $\tau = \sigma^2$.

(i) Find the Jeffreys' prior for (μ, β, τ).

(ii) Find the corresponding posterior marginal distribution of τ.

(iii) Find the corresponding posterior joint distribution of (μ, β). Does anything seem "off"? (Think about the classical maximum likelihood approach.)

Problem 2

Censored data: Suppose that $X_1, \ldots, X_n$ are i.i.d. random variables following an exponential distribution with unknown rate parameter λ (that is, $f(x) = \lambda \exp(-\lambda x)$). Consider the censoring model in which the X_i's are (possibly) censored by constants c_i. That is, we do not necessarily observe the X_i's. Instead, our data consists of pairs (Z_i, δ_i), $i = 1, \ldots, n$, where $Z_i = \min(X_i, c_i)$ and $\delta_i = I(X_i \leq c_i)$.

(a) Suppose that the prior distribution for λ is the Gamma distribution with parameters α and β (that is, $f(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}\exp(-\beta\lambda)$ for $\lambda > 0$. Find the posterior distribution of λ.

(b) Find the posterior distribution of λ when the Jeffreys' prior is used. Is the posterior always proper?

(c) For (a) and (b), describe how to find a 95% posterior probability interval for $\exp(-\lambda t_0)$ for a fixed constant $t_0 > 0$.