

Homework 4

Due Wednesday, February 3rd, 2016

Prevision

Let X be a bounded random quantity. The *prevision* of X for a subject A is the value $P(X)$ such that A is ready to accept any bet with gain $c(P(x) - X)$, where c is arbitrary, positive or negative, and at the choice of an opponent.

The set of previsions of subject A is *coherent* if there is no combination of bets that he has committed to accept for which the gains are all uniformly negative. That is, if X_1, X_2, \dots, X_n are random quantities, their previsions $P(X_1), P(X_2), \dots, P(X_n)$ have to be such that there is no linear combination

$$Y = c_1(X_1 - P(X_1)) + c_2(X_2 - P(X_2)) + \dots + c_n(X_n - P(X_n))$$

with $\sup(Y)$ negative or $\inf(Y)$ positive.

Show that the following statements are true for a coherent prevision:

1. If $X \leq 0$, then $P(X) \leq 0$;
2. if $Y = cX$, with $c \in \mathbb{R}$, then $P(Y) = cP(X)$;
3. if $X = c \in \mathbb{R}$ constant, then $P(X) = c$;
4. if X, Y and $X + Y$ are bounded random quantities, $P(X + Y) = P(X) + P(Y)$.

Scoring rules

A scoring rule is called *proper* when the maximum benefit is obtained by stating one own's probability evaluation sincerely. Let 1_E take on value 1 if the event E is true and 0 otherwise, let π be the probability of E declared by the subject, and let p be the subject truthful evaluation of the probability of E . Consider the two following loss that can be used to define scoring rules.

1. $\text{Loss}_1 = (1_E - \pi)^2$
2. $\text{Loss}_2 = |1_E - \pi|$

Show that 1. leads to a proper scoring rule, while 2. does not. What is π going to be for a scoring rule based on Loss_2 ?

Elections

You are going to need the collaboration of 10 friends for this. To each of these friends you are going to ask to name the fair price p for a bet that gives 1\$ if Hilary Clinton wins the primaries and 0 if she does not. Explain to them that they will choose the price, and then you will flip a coin: if the coin turns up head you will play the casino (pocket p and be ready to pay 1\$ if she wins), if it turns up tail, your friend will be the casino. Record all the declared prices and keep track of your duties to your friends. Against which of these friends you would think it is advantageous to play as a casino?