1. LET X_i $1 \le i < \infty$, Y_j $1 \le j < \infty$ BE BINARY RANDOM VARIABLES. SUPPOSE THAT, FOR FIXED VALUES OF THE X_i, THE Y_i ARE EXCHANGEABLE AND SIMILARLY, FOR FIXED VALUES OF THE Y_i, THE X_i ARE EXCHANGEABLE. USING THEOREMS STATED IN CLASS (OR OTHERWISE), SHOW THAT FOR ALL n, m a_i, b_i,

$$P(X_1 = a_1, \dots, X_n = a_n, Y_1 = b_1, \dots, Y_m = b_m) = \int_0^1\int_0^1 p_1^S (1-p_1)^{n-S}\, p_2^T (1-p_2)^{m-T}\, \mu(dp_1, dp_2)$$

WHERE $S = a_1 + \dots + a_n$, $T = b_1 + \dots + b_m$

2. SHOW THAT THE 'PARTIAL EXCHANGEABILITY' IN PROBLEM 1 IS VERY DIFFERENT THAN ASKING THAT <u>MARGINALLY</u>, THE X_i AND Y_j ARE EXCHANGEABLE (HINT, THIS IS EASY, WE ARE ASKING FOR A SIMPLE COUNTER EXAMPLE.

3. (A FINITE VERSION OF LAPLACE'S LAW OF SUCCESSION). SUPPOSE FIRST THAT $X_1, \dots, X_n$ ARE EXCHANGEABLE BINARY VARIABLES THAT ARE THE START OF AN INFINITE EXCHANGEABLE SEQUENCE WITH A UNIFORM PRIOR. OBSERVING S SUCCESSES OUT OF n, SHOW THAT THE PROBABILITY OF SUCCESS ON TRIAL $n+1$ IS $(S+1)/(n+2)$. NOW SUPPOSE THAT THE SAMPLE CAN ONLY BE EXTENDED TO $n+h$ FOR $h \ge 1$ FIXED. WE SHOWED THAT THE LAW OF X_i $1 \le i \le n+h$ CAN BE REPRESENTED AS A MIXTURE OF $n+h+1$ 'URN MEASURES'. PUT A UNIFORM PRIOR OVER THE URNS $(1/(n+h+1))$. SHOW THAT, GIVEN S SUCCESSES OUT OF THE FIRST n, THE CHANCE OF SUCCESS IS $(S+1)/(n+2)$ (NO MATTER WHAT h IS).