Stats 270/370 **Winter 2016**

Homework 7

Due Wednesday, February 24th, 2016

Multivariate Gaussian

Let $X \sim \mathcal{N}(\mu, \Sigma)$ be a $k \times 1$ random vector distributed as a multivariate gaussian with mean μ and variance covariance Σ. Let both μ and Σ be unknonw.

1. Write out a candidate conjugate prior for (μ, Σ) motivating your choice.
2. Derive the posterior joint distribution of μ and Σ associated to the prior you propose: is it in the same family of the prior?
3. What is the marginal posterior distribution for the vector of means μ?

Collinearity

Consider the standard linear regression model

$$y = X\beta + \epsilon$$

with y an $n \times 1$ vector of observations on the dependent variable, X an $n \times k$ matrix of independent variables, β a $k \times 1$ vector of parameters, and ϵ an $n \times 1$ vector of independent gaussian errors with mean zero and known variance σ^2. Let $n > k$ and assume that $\text{rank}(X) < k$ and carry out Bayesian inference on β.

What happens if $n > k$?

How to use the posterior distribution for point estimates

Let $p(\theta)$ be a distribution that describes your current beliefs on the parameter θ. Consider the following loss functions:

1. $\mathcal{L}_2(\theta, \hat{\theta}) = (\theta - \hat{\theta})^2$
2. $\mathcal{L}_1(\theta, \hat{\theta}) = |\theta - \hat{\theta}|$, with $|x|$ the absolute value of x.
3. $\mathcal{L}_0(\theta, \hat{\theta}) = 0$ if $|\theta - \hat{\theta}| < \epsilon$ and 1 otherwise (with ϵ an arbitrarily small positive number).

Show that the mean, the median, and the mode of $p(\theta)$ minimize the expected values of the loss functions respectively in a)-c), when the expectation is taken with respect to $p(\theta)$ and the minimization is done with respect to $\hat{\theta}$.