

# Stats 270/370 Homework 7

Due Friday, Mar. 4

## Problem 1

*A failure mode of Gibbs sampling.* In this exercise we investigate the effects of a bimodal posterior on the performance of Gibbs sampling. Suppose we have a statistical model with two-dimensional parameter  $\underline{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$ , and say  $\underline{\theta}$  has the following posterior distribution:

$$\underline{\theta} \mid \text{data} \sim \frac{1}{2}N(\mu_1, \Sigma) + \frac{1}{2}N(\mu_2, \Sigma).$$

Consider using Gibbs sampling to generate a sample from this posterior. Given the current state  $\underline{\theta}^{(t)}$ ,  $\underline{\theta}^{(t+1)}$  is generated through the following scheme:

1. Sample  $\theta_1^{(t+1)}$  from  $p(\theta_1 \mid \theta_2^{(t)}, \text{data})$
2. Sample  $\theta_2^{(t+1)}$  from  $p(\theta_2 \mid \theta_1^{(t+1)}, \text{data})$

Derive the Gibbs updates and implement the sampling procedure using your favorite software package. Run your procedure several times using  $\mu_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ,  $\mu_2 = \begin{pmatrix} a \\ a \end{pmatrix}$ , and  $\Sigma = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ . Try using small and large values for  $a$ , for example  $a \in \{0, 1.5, 10\}$ . Compare the Gibbs sampling results to true samples drawn iid from the posterior and comment on your findings. Repeat the experiment using  $\Sigma = \begin{pmatrix} 1 & 0 \\ 0 & \sigma^2 \end{pmatrix}$  for  $a$  large and  $\sigma \approx a$ ; comment on the results.